Interactive Cosmology Visualization Using the HUDF

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ABSTRACT

We have developed a Java[3]-based teaching tool, "Appreciating Hubble at Hyperspeed" ("AHaH"), intended for use by students and instructors in beginning astronomy and cosmology courses, which we have distributed via the World Wide Web[4]. This tool lets the user hypothetically traverse the Hubble Ultra Deep Field in three dimensions at over $\sim 500 \times 10^{12}$ times the speed of light, from redshifts $z = 0$ to $z = 6$. Users may also view the Universe in various cosmology configurations and two different geometry modes – standard geometry that includes expansion of the Universe, and a static pseudo-Euclidean geometry for comparison. In this paper we detail the mathematical formulae underlying the functions of this Java application, and provide justification for the use of these particular formulae. These include the manner in which angular sizes of objects are calculated in various cosmologies, as well as how the application's coordinate system is defined. We also briefly discuss the methods used to select and prepare the images in the application, the data used to measure the redshifts of the galaxies, and the qualitative implications of the visualization – that is, what exactly users see when they "move" the virtual telescope through the simulation.

Subject headings: Data Analysis and Techniques

1. Introduction

In beginning astronomy courses, many non-science majors appear to have a significant lack of understanding – even after taking the introductory courses – of basic concepts such as wavelength, electro-magnetic spectrum, speed-of-light, look-back time, redshift, and expansion of the Universe. We believe this lack of concept acquisition or retention represents a significant shortcoming of the currently available teaching tools. While pictures, figures, and other static media are certainly effective at communicating many concepts, they tend to be poor at showing effects in three dimensions or that evolve over time. Since virtually all cosmological effects require very large time or distance scales to become apparent, a different teaching medium is preferable.

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"Appreciating Hubble at Hyper-speed" is an educational tool that aims to address these issues of concept acquisition and retention by providing a visual and interactive learning medium. The project uses data from the HST Cycle 12 Project "GRAPES" (Grism-ACS Program for Extragalactic Science; Pirzkal et al. 2004) to build a redshift-sorted database of over 5000 galaxies within the Hubble Ultra Deep Field (HUDF). These galaxies range from redshift $z \approx 0.05$ to $z \approx 6$, spanning nearly 90% of the history of the Universe (Yan & Windhorst 2004; Bouwens et al. 2006). Since these data represent the deepest optical image of the Universe ever obtained, they are thus uniquely suited to students understand the effects of the expanding Universe.

2. Data Selection and Preparation

We first created a custom-balanced RGB version of the HUDF image. While the image provided in the original press releases would have been adequate, it has the undesirable characteristic that very bright areas, such as bulges in large spirals, appear burnt-out and lack fine detail. The raw HUDF data consist of B-, V-, i'-, and z'-bands (Beckwith et al. 2006), so we created a three-channel color image by first combining the B- and V-bands, applying weights based on the sky SNR[5]. We then used the algorithm developed by Lupton et al. (2004) to create the combined RGB image, with the combined $B+V$-bands as the blue channel, the i'-band as the green channel, and the z'-band as the red channel[6]. Besides showing more detail in bright areas, this method has the added benefit that an object with a specified astronomical color has a unique color in the composite RGB image. A comparison of the original STScI color images and our prepared images is shown in Figure 1. The full HUDF image using this color preparation technique is also available as an interactive map on the World Wide Web[7].

The galaxies represented in the AHaH application were i'-band selected using `SExtractor` with a detection threshold of $\sigma = 1.8$ above sky. The i'-band dropouts of Yan & Windhorst (2004) were later added by hand. We then created color JPEG "stamp" images for each individual object, using the `SExtractor`-generated segmentation map to mask as black any pixels outside the detected source. These "stamps" were then converted pixel-for-pixel to PNG images, which employ a lossless compression algorithm – no image quality was thus lost. We then developed a transparency map based on each pixel's brightness, which was saved into the PNG alpha channel. The resulting images can thus be displayed as semi-transparent, allowing objects in the distance to show through the dim regions of objects in the foreground.

Photometric redshifts for the galaxies were measured with `HyperZ` (Bolzonella, Miralles, & Pelló

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[5] 0.765 weight in V and 0.235 weight in B

[6] The channels were first scaled as follows, proportional to the data zero points – Red: 716.474, Green: 345.462, Blue: 254.449

[7] http://wwwgrapes.dyndns.org/udf_map/udfmain_low.html

2000), using a combination of the original HST-ACS four-band ($BVi'z'$) data from the HUDF, along with J- and H-band data from HST-NICMOS (Thompson et al. 2005). We have supplemented the photometric redshifts with spectro-photometric redshifts measured by Ryan et al. (2007), which incorporate the aforementioned $BVi'z'JH$ data as well as grism spectra from GRAPES (Pirzkal et al. 2004), U-band observations from CTIO-MOSAIC II, and K_s-band data from VLT-ISAAC. For a summary of all these data and the quality of the spectro-photometric redshifts, see Ryan et al. (2007). When available we have chosen to use the more reliable spectro-photometric redshifts.

3. Development of Formulae

As previously discussed by Wright (2006), there are a number of different methods for calculating distances in cosmology. For our purposes, the most meaningful of these is the comoving radial distance, D_R, representing the spatial separation of an object and an observer with zero peculiar velocity at a common time. This distance takes into account the expansion of the Universe, and so is more useful when dealing with distances on very large scales (and thus very large look-back times), as is the case with galaxies in the HUDF. Henceforth, we shall adopt the convention of referring to the comoving radial distance from Earth to a galaxy as D_R, and the comoving coordinate distance between two arbitrary points in the coordinate system as r_{ij}.

We also wish to calculate the angular sizes of objects as they would be observed from redshifts other than zero. To do so, we need a formula for the angular size distance, D_A. That is, the distance which satisfies the equation $d = \theta D_A$ for an object with transverse diameter d subtending an angle θ in the field of view. In a simple Euclidean space, this is the same as the radial distance, but again we must take into account the expansion (and possible curvature) of the Universe, so we must use a separate equation in the AHaH tool.

Additionally, we need to consider how we wish to define the coordinate system for the objects within the Java tool. Although we have very deep HST imaging data that allow us to show how the Universe has changed over time, all of these data were collected at a common time (2003/2004). Moreover, the principal distance measure that we have available, D_R, also assumes a common time. Thus the most sensible coordinate system is one with three spatial dimensions that makes all calculations for a common time, viz. when the data were collected. We can then contract the distances in this “comoving coordinate system” as necessary to simulate observations from redshifts other than zero. The question remains of how we should derive such coordinates from the data that we have in such a way that they will be useful to us – this is discussed in § 3.3 below, prior to deriving the equations.

3.1. Comoving Radial Distance

To begin, we need the comoving radial distance, D_R, from the Earth to an object at redshift z, derived from the Robertson-Walker metric, as discussed previously, e.g., Longair (1998, Ch. 7), Ryden (2003, Eq. 6.8), and Wright (2006, Eq. 6). We express this as the integral:

$$D_R = \int \frac{c \cdot dt}{a} = \int_{\frac{1}{1+z}}^{1} \frac{c \cdot da}{a\dot{a}} = \frac{c}{H_0} \int_0^z \frac{dz}{(1+z)\dot{a}}, \tag{1}$$

where the scale factor $a = 1/(1+z)$. The derivative of a with respect to time, $\dot{a}$, is given by the expression:

$$\dot{a} = (\Omega_M/a + \Omega_R/a^2 + \Omega_\Lambda \cdot a^2 + \Omega_K)^{1/2}, \tag{2}$$

where Ω_M, Ω_R, Ω_Λ, and Ω_K are energy density parameters, corresponding to the fractions of the Universe's total average energy density that are attributable to matter, radiation, dark energy, and the curvature of the spatial geometry, respectively. Note that it is assumed these are the only meaningful contributions to the total energy density – that is: $\Omega_M + \Omega_\Lambda + \Omega_R + \Omega_K = 1$.

We evaluate this integral in steps of 0.05 in z from $z = 0$ to $z = 20$ to create a look-up table, interpolating linearly to find the value for any arbitrary redshift. This is because we must make the calculation frequently and for many objects, so computing the integral manually every time would be computationally prohibitive. The resultant error in this method is generally small enough that it translates to less than one pixel's difference even on high-resolution displays, so it can safely be ignored for the purposes of the application. We evaluate the integral using the simple midpoint method, which may not be the optimal solution, but was simple to implement and adequately efficient. As with the linear interpolation, higher accuracy numerical integration would result in less than one pixel's difference when displayed.

3.2. Angular Size Distance

To develop the angular size distance, D_A, we first need the comoving tangential distance, D_T, of an object at redshift z_j as measured by an observer at redshift z_i. This distance is given by the formula:

$$D_T(z_i, z_j) = \begin{cases} \Re' \sin(r'_{ij}/\Re') & \text{if } \Omega_K < 0 \\ r'_{ij} & \text{if } \Omega_K = 0 \\ \Re' \sinh(r'_{ij}/\Re') & \text{if } \Omega_K > 0 \end{cases}, \tag{3}$$

where $\Re'$ is the radius of curvature of the spatial geometry at redshift z_i and r'_{ij} is the value of the comoving coordinate distance at the same redshift (Longair 1998; Wright 2006). These correspond to the cases where the spatial geometry is spherically curved, flat, and hyperbolically curved, respectively. Recalling that both r'_{ij} and $\Re'$ scale as $1/(1+z_i)$, and that $\Re = (c/H_0)/\sqrt{|\Omega_K|}$, we may rewrite D_T as:

$$D_T = \frac{\delta_{ij}}{1+z_i} r_{ij}, \tag{4}$$

where δ_{ij} is simply some function of r_{ij}.

We first define an intermediate quantity U_{ij}[8], representing the argument of sin and sinh in Equation (3).

$$U_{ij} = r'_{ij}/\Re' = r_{ij}/\Re = (H_0/c)\sqrt{|\Omega_K|}r_{ij} \tag{5}$$

Now substituting U_{ij} into Equation (3) above, we get the following expression for δ_{ij}:

$$\delta_{ij} = \begin{cases} \frac{\sin(U_{ij})}{U_{ij}} & \text{if } \Omega_K < 0 \\ 1 & \text{if } \Omega_K = 0 \\ \frac{\sinh(U_{ij})}{U_{ij}} & \text{if } \Omega_K > 0 \end{cases} \tag{6}$$

Note that δ_{ij} expressly depends upon r_{ij}. The case where $\Omega_K = 0$ comes from the limit of both $\sin(U_{ij})/U_{ij}$ and $\sinh(U_{ij})/U_{ij}$ as $\Omega_K \to 0$ – one may observe that Equation (4) then simplifies to $r_{ij}/(1+z_i)$, which is precisely r'_{ij} as in Equation (3).

Thus, using the equation relating the angular size distance and the tangential distance as developed by Longair (1998, Eq. 7.50), the angular size distance from redshift z_i to z_j is given by:

$$D_A = \frac{1+z_i}{1+z_j} D_T = \frac{\delta_{ij}}{1+z_j} r_{ij} \tag{7}$$

3.3. Comoving Coordinate System

Now that we have developed formulae for D_R and D_A, we can consider the best way to create a coordinate system for the Java application. The data we start with are the redshift of an object (with which we can calculate D_R) and four angles: the object's angular size (from the height and width of its image) and the angular separation between the object and the x and y axes, which we define as lines going through the center of the original image. These angles are calculated by taking the corresponding size in pixels and multiplying by the scale in arcsec/pixel of the original HST image[9].

We would like to use this information to create a coordinate system with the original telescope position at the origin. In a Euclidean space this would present no problem, but we have already remarked that the *observed* angles are not the same in an expanding Universe as they would be in a Euclidean space. Further, it would be desirable for the Euclidean coordinate distance to correspond to the comoving radial distance, as this would make calculations significantly simpler. We can accomplish this, but when we create coordinates for each object as such, we need to "correct" the

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[8]Though defining U_{ij} in this way means we must multiply δ_{ij} by r_{ij} to obtain D_T, it decreases the total number of calculations that we must make within the Java application, since we may calculate U_{ij} once per object and re-use it.

[9]0".03 per pixel

angles. That is, we want a "Euclidean angular size" associated with a certain observed angular size. We will call this θ_E. An object's angular size is related to its physical transverse diameter, d, by the equation:

$$d = \theta D_A = \theta \frac{\delta_{ij}}{1+z_j} r_{ij} = \theta_E \frac{1}{1+z_i} r_{ij} \tag{8}$$

Note that in the Euclidean case we must contract r_{ij} by a factor of $1/(1+z_i)$ to get the comoving distance from z_i to z_j as measured from z_i (r'_{ij} in Equation (3) above). This is because the proper spatial separation in the current epoch has been stretched by the Universe's expansion, so from redshift z_i it must be scaled appropriately.

Thus canceling r_{ij}, we get the following expression for θ_E:

$$\theta_E = \theta \delta_{ij} \frac{1+z_i}{1+z_j} \tag{9}$$

In our initial data z_i is simply zero, we create coordinates (X, Y, Z) for an object like:

$$X = \sin\left(\frac{\delta_{ij}\theta_X}{1+z}\right) \cos\left(\frac{\delta_{ij}\theta_Y}{1+z}\right) D_R, \tag{10}$$

and similarly for Y and Z. We have thus developed a coordinate system of X, Y, and Z in Mpc with the original telescope position at the origin.

3.4. Simulating Observations From Vantage Points Other Than $z\!=\!0$

Now, when we "move" the camera, we do so by some X_c, Y_c, and Z_c in the coordinate space. By construction, the distance measure here is just the Euclidean coordinate distance:

$$D_E = ((X - X_c)^2 + (Y - Y_c)^2 + (Z - Z_c)^2)^{1/2} \tag{11}$$

Now to determine where to display an object after we have "moved" the camera, we use the distance calculated with Equation (11) and the Euclidean angular size. Using the object's redshift, z_o, and the camera's user-defined redshift, z_c, we rearrange Equation (9) to get:

$$\theta = \theta_E \frac{1+z_o}{\delta_{ij}(1+z_c)} \tag{12}$$

In this case, θ_E is a quantity that we must calculate from our coordinates in the usual Euclidean way.

For an object's angular size it is even simpler than for its (X, Y, Z) position, since we do not have to manually calculate θ_E. We know that in the Euclidean case:

$$d = \theta_0 D_R = \theta_E D_E, \tag{13}$$

where θ_0 is the Euclidean angular size from redshift zero, and D_E is the coordinate distance from the camera to the object. We then solve for θ_E and substitute into Equation (12) to obtain an expression for the desired angular size θ as observed from z_c:

$$\theta = \theta_0 \left(\frac{D_R}{D_E}\right) \frac{1+z_o}{\delta_{ij}(1+z_c)} \tag{14}$$

4. Standard Display Mode

While some might argue that the above equations speak for themselves, we believe it is very instructive to consider the qualitative implications of their use – that is, a description of what exactly we see when we "move" the camera in the Java application. For the sake of completeness, we will also detail a number of cosmological effects that have been omitted from the application due to technical limitations. An example of the standard display mode is shown in Figure 3.

When we move the camera to a certain position in the HUDF data cube, we are in general viewing the Universe as it would appear from that point and at that redshift. We must qualify this statement by noting that the simulation accounts *only* for cosmological effects of changing the camera position – no dynamical, lensing, evolutionary, or other effects are simulated. In this sense, AHaH thus truly, though hypothetically, allows the user to travel through the Universe at "hyper-speed."

The somewhat counterintuitive relationship between an object's angular size and its redshift is readily apparent in the standard display mode. If a user slowly increases the redshift of the camera, high redshift objects will begin to decrease in angular size and move toward the center of the display, eventually reaching a minimum angular size and then increasing. Also visible are the effects of galaxy evolution and merging over time. For example, when viewing the Universe from redshift $z = 0.5$ as in Figure 3, there are many large spiral and elliptical galaxies visible. However, when viewing the Universe from redshift $z = 1.5$ as in Figure 4, the screen is dominated by small and compact blue galaxies.

It should be noted that the application does not make calculations for cosmological surface brightness dimming or changes in color due to redshift or spectral evolution. While certainly feasible to simulate, performing such image manipulation techniques on large numbers of galaxies in real-time is currently too difficult for consumer computers. Moreover, we must also recall that the HUDF data are limited in both magnitude and effective horizon by what could be observed from low Earth orbit. When we view the data from redshifts other than zero, we would expect to see more galaxies overall – including fainter galaxies – than are represented in the current HUDF data. We could choose to simulate these objects as extensions of our data set if we desired, but we felt this would not be particularly instructive, and could lead to potential confusion. Moreover, such simulations have a high degree of uncertainty and, by significantly increasing the size of the data set, would add prohibitively to the computation times. Likewise, we have chosen not to simulate galaxies outside of the original field, which would of course enter the camera's field of view as the user pans around.

5. Static Geometry Mode

When a user presses the "G" key in the Java tool, they are told that they are viewing the simulation with "static geometry" turned on. What this means specifically is that angular sizes as derived above are no longer affected by the scale factor or curvature of the Universe – after we develop our original coordinates, as in Equation (10), all calculations for angles are simply done with $\theta = \theta_E$. This has the visual effect of all galaxies appearing smaller and closer to the center of the viewport, since all initial angles have been contracted by a factor of $(1 + z)$ (when Ω_K is zero). In this static case, galaxies will also simply increase in angular size as we approach them, as opposed to the angular sizes of high-redshift objects in the real WMAP Universe, which decrease, reach a minimum, and then increase as the camera's redshift increases.

This static mode of viewing the simulation has no physical analogue – it is simply meant to convey to the user that there are non-Euclidean aspects of the Universe's geometry, and that the angular sizes we observe in the present have been made larger due to the Universe's expansion. One should note that this display mode only considers expansion as it relates to angular size – the comoving radial distance is still calculated using the redshift and curvature factors that would not be present in a strictly Euclidean Universe. That is, in the static display mode, we assume that the Hubble Law distance, $D = v/H_0 = (c/H_0)z$, is simply a Euclidean distance unrelated to expansion. This is primarily because our method of calculating the comoving radial distance relies upon redshift, which is a phenomenon specific only to an expanding Universe, and is therefore the only way we could calculate distances for all galaxies.

6. Conclusion

We believe that this software provides students and instructors with an unprecedented ability to interactively visualize many of the effects of an expanding Universe, among its other capabilities. The application should help clarify these concepts, and allow students to develop a deeper intuitive understanding of the material. Certain cosmological effects – such as bandpass shifting, k-correction, surface brightness dimming, gravitational lensing, and the effects of the magnitude limit and object sizes on the sample completeness limit – have been omitted due to computational limitations, but we believe these to be inessential for the understanding of the included effects. For a discussion of most of these effects, see e.g. Windhorst, Hathi, Cohen, & Jansen (2007).

For the convenience of those who wish to see or modify the particular implementation of the above formulae within the Java software, we have provided source code with the standard distribution of the tool. It is included in the src/ directory of ahah.jar and may be extracted using the java `jar` utility or any zlib-compatible de-compressor such as `unzip`. The tool may be downloaded from the AHaH website[10].

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A. AHaH User Manual

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[10]http://www.asu.edu/clas/hst/www/ahah/

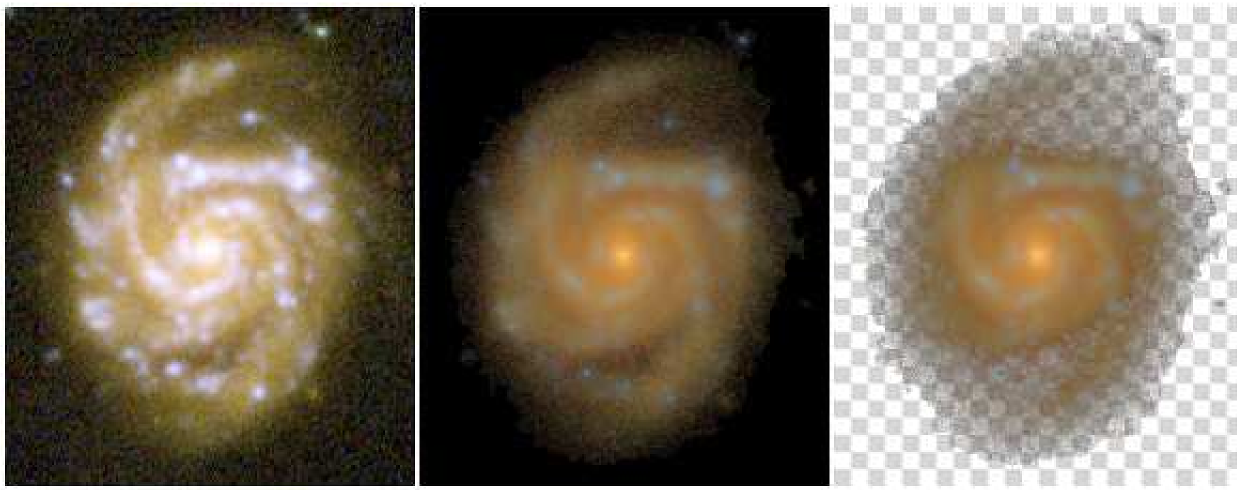

Fig. 1.— A comparison of three images of HUDF galaxy 7556. The left image is that from the original STScI release, clearly showing the bright, burnt-out knots characteristic of the standard logarithmic image stretch. The center image is our prepared image using the arcsinh stretch described by Lupton et al. (2004), as it appears in the AHaH application. The right image is our prepared image against an artificially imposed chessboard pattern, showing the included transparency. Note that pixels outside the source are all completely transparent, since they have been removed entirely using the `SExtractor` segmentation map.

Fig. 2.— Our prepared images of three galaxies from the HUDF, using the arcsinh stretch described by Lupton et al. (2004). Shown are galaxy 3180 (left), galaxy 5805 (center), and galaxy 6974 (right).

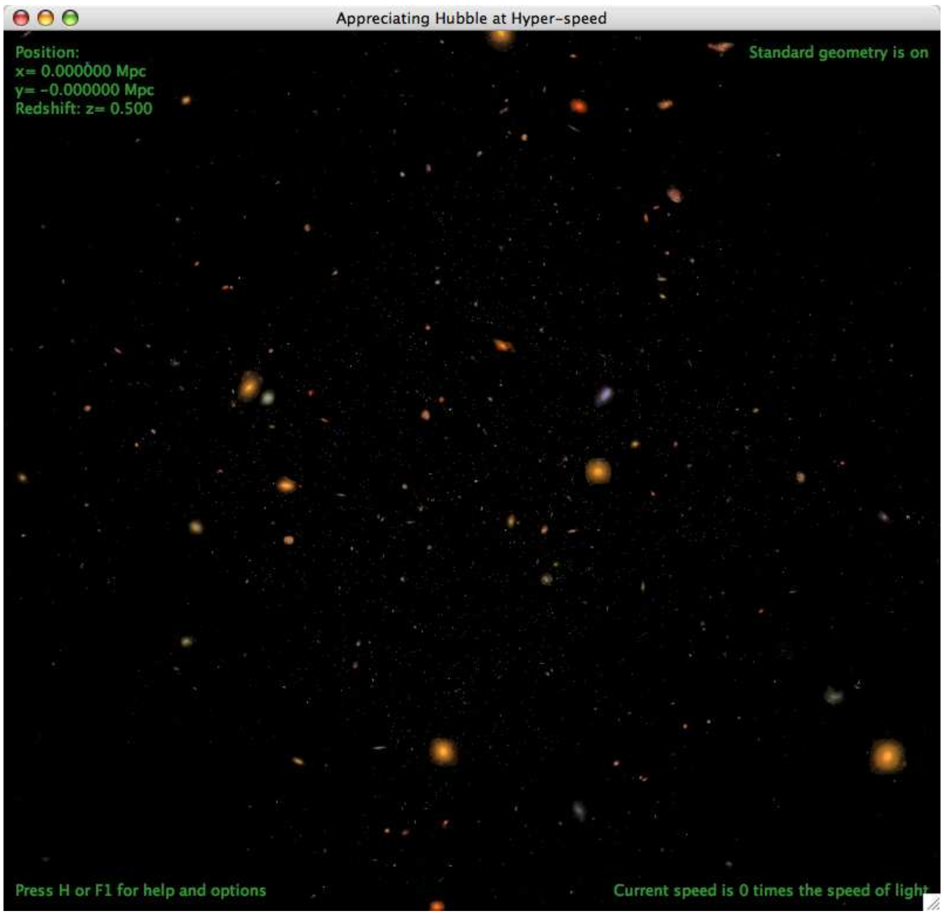

Fig. 3.— The HUDF data as viewed from redshift $z = 0.5$ in the AHaH application, using standard geometry mode, which properly calculates angular sizes. Note how the image is dominated by luminous red early-type galaxies.

Fig. 4.— The HUDF data as viewed from redshift $z = 1.5$ in the AHaH application, using standard geometry mode. Note how this image is dominated by blue irregular and merging star-forming galaxies.